LECTURE 6

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Exercise 38.

$$\lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}((\frac{3k}{n})^2-\frac{3k}{n})$$

$$\begin{aligned}
& \lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}((\frac{3k}{n})^2-\frac{3k}{n}) \\
= & \lim_{n\to\infty}\frac{3}{n}\sum_{k=1}^{n}\left(\frac{9k^2-3kn}{n^2}\right) \\
= & \lim_{n\to\infty}\frac{9}{n^3}\sum_{k=1}^{n}\left(3k^2-kn\right) \\
= & \lim_{n\to\infty}\frac{9}{n^3}\left(\sum_{k=1}^{n}3k^2-n\sum_{k=1}^{n}k\right) \\
= & \lim_{n\to\infty}\frac{9}{n^3}\left(\frac{3n(n+1)(2n+1)}{6}-n\frac{n(n+1)}{2}\right) \\
= & \lim_{n\to\infty}\frac{9}{n^3}(\sim n^3 - \sim\frac{n^3}{2}) \\
= & \frac{9}{2}
\end{aligned}$$

Exercise 42. f: infinitely diff on $\mathbb{R}$
$f(0)=1, f'(0)=1, f''(0)=2, |f'''(x)|<b$ for some $b, \forall x\in[0,1]$
$f(1)<5$
Question: max value for b?
$f's$ Taylor series/approx
$f=1+1x+1x^2+?x^3+\cdots$
$|f'''(0)|<b \Rightarrow |a_n|<\frac{b}{6}$
$f(1)=$ sum of all coefficients
$1+1+1+\sum_{n=3}^{\infty}a_n<5$
$|sum_{n=3}^{\infty}a_n n(n-1)(n-2)x^{n-3}|<b$
Alt approach

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$f(x) = 1 + x + x^2 + E_2(x)$

$$\begin{aligned} & E_2(x) \\ = & \int_0^x \frac{f'''(t)}{2!}(x-t)^2 dt \\ \leq & \int_0^x \frac{b}{2}(x^2 - 2xt + t^2) dt \\ = & \frac{bx^2}{2}t - \frac{bxt^2}{2} + \frac{bt^3}{6}|_0^x \\ = & \frac{bx^3}{6} \end{aligned}$$

$|E_2(x)| \leq \frac{bx^3}{6}$
$f(1) = 1 + 1 + 1 + E_2(1) < 5 \Rightarrow E_2(x) < 2 \Rightarrow b < 12$

Exercise 44. $x \in \mathbb{R}, P(x)$ polynomial
Q:$\lim_{h \to 0} \frac{P(x+3h)+P(x-3h)-2P(x)}{h^2}$

L'H

$$\begin{aligned} = & \lim_{h \to 0} \frac{3(p'(x+3h) - 3P'(3-3h)}{2h} \\ & L'H! \\ = & \lim_{h \to 0} \frac{9P''(x+3h) + 9P''(x-3h)}{h^2} \\ = & 9P''(x) \end{aligned}$$

Exercise 49. for X a random variable with probability distribution

$$f(x) = \begin{cases} \frac{3}{4}(1-x^2) & x \in [-1,1] \\ 0 & o/w \end{cases}$$

what is standard deviation?
$\sigma(x)\sqrt{E(x^2) - (E(x))^2}$

$$\begin{aligned} E(x) &= \int x f(x) dx \\ &= \int_{-1}^{1} x \frac{3}{4}(1-x^2) dx \\ &= \frac{3}{4}\int_{-1}^{1}(x - x^3) dx \\ &= 0 \end{aligned}$$

$$\begin{aligned} E(x^2) &= \int x^2 f(x)dx \\ &= \int_{-1}^{1} x^2 \frac{3}{4}(1-x^2)dx \\ &= \frac{3}{4}\int_{-1}^{1}(x^2-x^4)dx \\ &= \frac{3}{4}\,(\frac{1}{3}x^3-\frac{1}{5}x^5)\Big|_{-1}^{1} \\ &= \frac{1}{5} \end{aligned}$$

$$\sigma(x) = \sqrt{\frac{1}{5}-0} = \frac{1}{\sqrt{5}}$$

Exercise 62. Let $X = [0,1]$, with the topology induced by the basis sets $\{[a,b)|a,b \in \mathbb{R}\}$
topology: a collection of sets that you call open
$\cup_{any}$ open=open; $\cap_{finite} open =$ open
basis: all open sets = a union of $\{[a,b)|a,b \in \mathbb{R}\}$.
Q: is X
I connected?
II Hausdorff?
III compact?
X is disconnected if can write $X = A \cup B, A, B$ open and disjoint
Hausdorff: $\forall$ any 2 points a, b in X, can find A, B disjoint and open, $a \in A, b \in B$
Compact: if $A = \cup$ open sets can pick finite from $\cup$ open sets and still have A.
I: $A = [0, \frac{1}{2}), B = [\frac{1}{2}, 2)$ check
II: pick small enough
III: No $[0, 1-\frac{1}{n})$